Choosing, creating and using story problems: Some helpful hints

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In this article Anne Roche describes some of the different types of story problems defined in the Cognitively Guided Instruction professional development program. Teachers will find the table reproduced on page 32 to be very helpful in designing word problems. Anne then gives some suggestions for improving the way division stories are used in the classroom setting.

As part of a professional learning program, 358 primary teachers were asked the following two questions in a questionnaire at the end of 2011:

1. How often do you create your own story problems for students to solve?
2. How often do you use story problems in a mathematics lesson that have *not been* created by you?

For each question, the teachers were prompted to choose from one of the following options: most lessons; once a week; once a month; once a term; and never.

The respondents taught the following year levels: 156 in Prep to Year 2, 105 in Years 3 to 4, and 97 in Years 5 to 6.

Figures 1 and 2 show the percentage of teachers who chose each option for the two questions.

The results showed that 72.6% of the teachers created story problems at least once a week and that 64.0% of the teachers use story problems *not* created by them at least once a week. Given the regularity with which teachers claim to use and create their own story problems, it is therefore of interest how well this is done.

This article describes some of the different types of story problems defined by the Cognitively Guided Instruction (CGI) teacher development program (Carpenter, Fennema, Franke, Levi & Empson, 1999), and discusses what constitutes a well-written story problem. The latter will be based on analysing teacher-created story problems

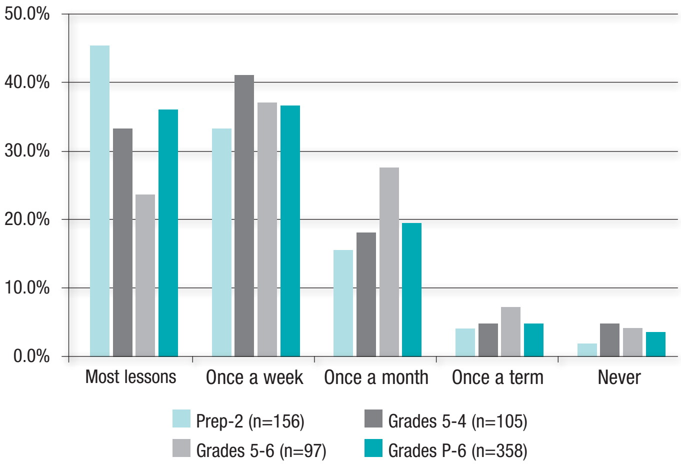

Figure 1. Frequencies in response to: "How often do you create your own story problems for students to solve?"

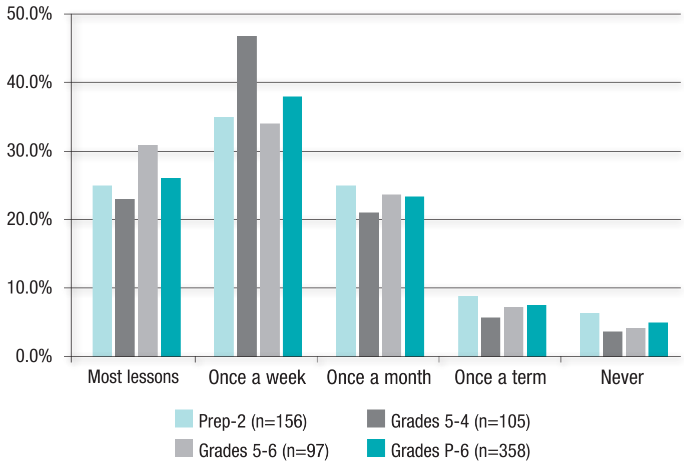

Figure 2. Frequencies in response to: "How often do you use story problems in a mathematics lesson that have not been created by you?"

from a large professional learning and research project. In particular, this paper focuses on story problems that teachers wrote to represent each form of division (i.e., partitive and quotitive models).

The CGI Program referred to these kinds of problems as 'word problems', but for the purpose of this paper, I will refer to them as 'story problems'. I classify story problems as a *type* of word problem. I define a story problem as a *closed* one-step problem that has the structure of two or more statements followed by a question. Solving such a story problem requires "decoding information about relationships and operations among known and unknown quantities" (Ben-Hur, 2006, p. 85). On the other hand, I define word problems more broadly, as including any mathematical problem written in a sentence or sentences. For example, word problems could include (but not exclusively) one or more steps, could be open-ended (Sullivan & Lilburn, 1997), could be categorised as tasks with contexts (Clarke & Roche, 2009), or could be extended investigations.

Cognitively guided instruction

CGI outlined a helpful classification scheme for describing different types of addition, subtraction, multiplication and division problems (the problem types for addition and subtraction are shown in Figure 3). In the case of multiplication and division, the problem types included Grouping/Partitioning, Rate, Price, and Multiplicative Comparison. For more information on different categorisations of multiplication and division problems, see Downton (2009).

The CGI program included around 75 hours of workshop time over three years and the mathematical content was based almost exclusively on word problems (Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996). Being able to create such appropriate problems was considered essential to the work of a CGI teacher.

The CGI team also described the development of children's strategies for solving story problems. Essentially children begin by modelling the action or relationship described in the problem, which over time gives way to counting strategies and then strategies that utilise derived number facts, and finally invented procedures. In terms of relative difficulty, problems that are difficult to model directly are generally more difficult to solve. In particular, the Start Unknown problems (e.g., Aisha had some little biscuits. She gave 5 to Lisa. Now she has 8 little biscuits. How many little biscuits did Aisha have to start with?) are difficult to model because the initial quantity is unknown and therefore cannot be represented. Garcia, Jimenez and Hess (2006) emphasised the importance of the position of the unknown quantity in a number sentence in affecting the difficulty of the problem.

Two important outcomes of the CGI program were that the teachers became more familiar with the full range of problem

types, many of which had not been used with their students before. Also, the teachers understood that some problems are harder than others, and therefore require more sophisticated strategies to solve.

Story problems provide a context for students to think about and model important mathematical ideas. In the early years, it is important that students are left to represent and solve story problems in a variety of ways without an over-emphasis on symbolic expressions.

Problem type	Result unknown	Change unknown	Start unknown
Join	Tom had 9 marbles. Jack gave him 5 more marbles. How many marbles does Tom have altogether?	Kate has 9 pencils. How many more pencils does he need to have 14 altogether?	Aisha had some birthday presents. At the party she got 5 more presents. Now she has 14. How many presents did Aisha have to start with?
Separate	Jack had 13 lollies. He gave 5 lollies to Jack. How many lollies does he have left?	Kate had 13 stickers. She gave some to Rose. Now she has 5 left. How many stickers did she give to Rose?	Aisha had some little biscuits. She gave 5 to Lisa. Now she has 8 little biscuits. How many little biscuits did Aisha have to start with?
	Whole unknown	**Part unknown**	
Part–part–whole	In the chicken pen there are 4 brown chickens and 8 white chickens. How many chickens are in the pen altogether?	In the fruit bowl there are 14 apples. 9 are red apples and the rest are green apples. How many green apples are in the fruit bowl?	
	Difference unknown	**Compare quantity unknown**	**Referent unknown**
Compare	Louise has 12 strawberries. Eva has 7 strawberries. How many more strawberries does Louise have than Eva?	Luke has 7 comic books. Paul has 5 more than Luke. How many comic books does Paul have?	Con has 12 trophies. He has 5 more trophies than his friend, Bob. How many trophies does Bob have?

Figure 3. CGI Classification of word problems for addition and subtraction (adapted from Carpenter, et al., 1999, p. 12).

In the context of this article, the value of the CGI work is that it provides a summary of different kinds of story problems and ways of structuring story problems which are both mathematically and grammatically correct.

Representing division: What can be learned from research?

Ball (1990) explained that "at its foundation, division has to do with forming groups" (p. 452). She distinguished the two groups as *forming groups of a certain size* (quotitive division) and *forming a certain number of groups* (partitive division). Research has shown that many school students, pre-service teachers and inservice teachers, have difficulty making sense of both forms of division, and in particular quotitive division, and its relevance to division by a fraction (see Roche & Clarke, 2009).

From 2008 to 2011, teachers who were part of the Contemporary Teaching and Learning Mathematics Project (CTLM[1]) were asked twice a year to complete a questionnaire about their *mathematical knowledge for teaching* (MKT; Hill, Schilling & Ball, 2004). In one item called *Division Stories,* they were asked to name each form of division, draw a simple picture, and write a story problem that represented each form of division for 12 ÷ 3. (For a complete description of this item, see Roche & Clarke, 2009). After three years of asking teachers to represent 12 ÷ 3 in these ways, it became clear that teachers were more likely to have a preference for and comfort with the partitive model. That is, they were more likely to be able to represent a whole

1 We acknowledge gratefully the support of the Catholic Education Office (Melbourne) and that of Gerard Lewis and Paul Sedunary in particular in the funding of this research.

number division using a sharing context; e.g., 12 stickers are shared equally among 3 children. How many stickers did each child get? They were much less likely to identify and describe the 'other' form (quotitive division). An example of quotitive division for 12 ÷ 3 is: "Kate drew 12 ladybugs and some leaves. She drew 3 ladybugs on each leaf. How many leaves did she draw?"

Results

At the end of 2011, 378 primary teachers (including those teachers who responded to the two questions at the beginning of this article), completed the item Division Stories. For the purpose of this paper I will focus only on the capacity of the teachers to write two story problems, one for each form of division for 12 ÷ 3. The results showed that: 29.1% of teachers wrote two well-worded story problems, one for each form of division; 39.4% wrote one; and 31.5% did not write any well-worded examples of a story problem for 12 ÷ 3. The most common error was writing a story problem that did not mathematically represent 12 ÷ 3. Two examples of this from teacher responses are:

- Mum makes 12 biscuits. Each child gets 4 biscuits. How many children are there?
- There were 3 enclosures and each holds 4 animals. How many animals can the zoo have?

These errors may have been the result of teachers' attempts to match a story problem to one of their simple pictures for 12 ÷ 3 (see Figure 4). Both incorrect story problems can correctly be represented by the simple picture that represents partitive division for 12 ÷ 3, but symbolically they represent the expressions 12 ÷ 4 and 3 × 4, respectively.

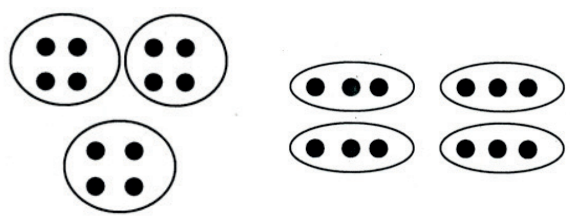

Figure 4. Simple pictures that represent partitive and quotitive division for 12 ÷ 3.

I need to make it clear that the CTLM teachers had less than one hour of professional learning on the two forms of division and had no professional learning on developing story problems or considering the different types of story problems as described earlier. However, what became evident from the teachers' attempts to write story problems were the many nuances that make a story problem clear and unambiguous, and therefore the complexity of creating one.

The role of the teacher when using story problems

When students are engaged in solving story problems, the role of the teacher is complex. Apart from carefully selecting from a variety of problem types, teachers should sequence the story problems in order to gradually build up the difficulty level. Teachers should also provide opportunities for students to talk or write about how they solve problems, in order to find out what they know and understand.

In a previous edition of this journal, Monroe and Panchyshyn (2005) provided advice for helping students engage with word problems, which I take to be equally relevant to story problems. They recommended that teachers:

- remember to teach the new vocabulary;
- assign only a few well-chosen problems and allow the time needed for students to explore the mathematics;
- adapt ready-made problems by changing the names to those of students in your class;
- provide sufficient context to help students to 'see' the problem mentally;
- use contexts familiar to students such as those from their own experiences or familiar story books; and
- encourage students to write their own story problems.

In a relevant study, van Klinken (2012) demonstrated the effectiveness of the use of schematic diagrams by Grade 3 students in synthesising information so that only the important structural elements remain.

Some further advice arising from this study

The following list provides some helpful hints or criteria for creating story problems that represent a division situation and story problems in general. The kinds of errors which emerged from teachers' responses to *Division Stories* led to the generation of these hints.

It should be stressed that the discussion assumes we are talking about writing a story problem to match a simple equation, involving one operation. Each hint is followed by examples of teacher responses that *did not* satisfy this criteria and an explanation why.

1. A story problem should be a closed problem (having only one possible answer).

- There are 12 lollies and 3 children. How many lollies does each child get?
- The class has 3 bug catchers. The children find 12 bugs. How many bugs should be placed in each bug catcher?

Each of these problems does not make it clear that the lollies and bugs are to be distributed equally and a child could be correct in stating that they were distributed in groups of, 2, 4 and 6 for example, or 3, 2 and 7, etc. thus providing several possible correct solutions.

2. A story problem should state a question.

- I have 12 apples and I share them with my three friends.
- I had 12 marbles and I put them into groups of 3.

To solve a story problem, the students need to determine the *unknown quantity*, but in these examples it is not clear what the unknown quantity might be, namely how many in each group or how many groups? In fact, there is nothing to solve.

3. A story problem (in the context discussed here) should state two known quantities.

- There are 12 lollies. How many would each person get?
- Twelve teachers were at school. They were split equally into each level. How many teachers in each level?

Each of these examples is missing important information about the size of one of the quantities.

4. A story problem should use appropriate mathematical terms for its context.

- I had 12 pens and I put them evenly into three containers. How many in each container?
- I have 12 stickers and I shared them between 3 people. How many do they get?

The first example inappropriately uses the word *evenly* instead of *equally*. Although the word *evenly* is used in everyday conversation to give a sense of equality, in the mathematical context its use is likely to confuse students who are trying to make sense of its use in terms of their experience with odd and even numbers. To emphasise this point, it is worth considering what sense a student might make of a question such as: "Can an odd number of apples be shared evenly?"

The second example is missing the word *each*. To make the problem clear it is important to ask, "How many do they *each* get?" or a student may correctly respond that they get 12 stickers altogether.

5. A story problem requires a context if students are to relate to it.

- In 12, how many groups of 3?
- Share 12 equally into 3 groups.

Neither example has a context to help children engage in the problem. A story problem by definition needs a story. The second example also does not state a question.

A number of the examples provided by the teachers had more than one criteria missing in a single story problem.

Conclusion

This advice may seem a little pedantic, but my experience has shown that the task of creating appropriate story problems for a given division expression is indeed difficult. It takes time and a commitment to make sense of a complex area of mathematics in a way that is helpful for students. While the advice has focused on story problems in particular, I suggest many of the points made in this article are useful for creating a range of word problems.

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References

Ball, D. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449–466.

Ben-Hur, M. (2006). *Concept-rich mathematics instruction: Building a strong foundation for reasoning and problem solving.* Alexandria, VA: Association for Supervision and Curriculum Development.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann and National Council of Teachers of Mathematics.

Clarke, D. M., & Roche, A. (2009). Using mathematical tasks built around 'real' contexts: Opportunities and challenges for teachers and students. *Australian Primary Mathematics Classroom, 14*(2), 24–31.

Downton, A. (2009). It seems to matter not whether it is partitive or quotitive division when solving one step division problems. In R. Hunter, B. Bicknell & T. Burgess (Eds), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education research Group of Australasia, Vol. 1, pp. 161–168). Palmerston North, NZ: MERGA.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403–434.

Garcia, A. I., Jimenez, J. E., & Hess, S. (2006). Solving arithmetic problems: An analysis of classification as a function of difficulty in children with and without arithmetic learning difficulties. *Journal of Learning Difficulties, 39*(3), 270–281.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11–30.

Monroe, E., & Panchyshyn, R. (2005). Helping children with words in word problems. *Australian Primary Mathematics Classroom, 10*(4), 27–29.

Roche, A., & Clarke, D. (2009). Making sense of partitive and quotitive division: A snapshot of teacher's pedagogical content knowledge. In R. Hunter, B. Bicknell & T. Burgess (Eds), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia Incorporated, pp. 467–474). Palmerston: MERGA.

Sullivan, P., & Lilburn, P. (1997). *Open-ended maths activities: Using 'good' questions to enhance learning.* Melbourne: Oxford University Press.

van Klinken, E. (2012). Word problem solving: A schematic approach in Year 3. *Australian Primary Mathematics Classroom, 17*(1), 3–7.